Reliability-Based Methods for Electric System Decision Making

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Abstract

The purpose of this paper is to develop a methodology that utilizes reliability-based optimization to solve complex electrical grid usage problems. With electrical power grids, as with many complex systems, complicated decisions must be made at both the local (user) and global (electricity provider) levels; all decision makers have independent, often conflicting, objectives, further complicating the decisions. In order to incorporate both levels of decision making (and resulting interaction effects between the decision makers), a reliability-based optimization approach can be utilized which incorporates local decision makers' preferences by enforcing probabilistic constraints on the overall optimization problem (e.g., sectors A and B need a particular amount of power and each sector has a different criticality level). This ensures that the optimized decisions made at the global level satisfy the basic requirements of the local decision makers (e.g., to deliver power to critical sectors). The uncertainty in this approach is incorporated through an efficient first order reliability method (FORM), an analytical approximation to failure probability calculation, rather than traditional, computationally expensive simulation-based methods (such as Monte Carlo sampling). Usefulness of this methodology is shown through several example problems.

1. INTRODUCTION

As demand increases nationwide for electrical power, the nation must look for intelligent approaches to managing electrical distribution. This requires the development of an electricity management approach that determines the optimal allocation of power to subsystems such that the cost of power is minimized.

In order to achieve this, complex electrical grid usage problems require the interaction of individuals at both local and global levels. At the local level, users expect power to be available on demand (i.e. with 100% reliability). At the global level, electricity providers are struggling to meet the demands of their customers in the most cost efficient manner possible. Thus, it benefits the electricity providers to have the minimum amount of power available so as not to waste electricity when it is not being used by customers. These objectives are inherently competing as maintaining electrical service availability for users is costly. Additionally, the criticality of some infrastructures (e.g. hospitals, police stations) requires greater certainty of power availability than the average user. Combining these factors results in a complicated decision making problem.

This paper develops a methodology for electric system decision-making at both a local and global level. It begins by discussing a basic problem formulation for local and global decision making to facilitate an interoperable electric grid. The detailed mathematical approach behind this methodology is then discussed. Sample problems employing this methodology are demonstrated. Finally some conclusions and recommendations for problem extensions are discussed.

2. PROBLEM FORMULATION

The goal of this methodology is to develop an approach for electric system usage which incorporates the needs of both local and global decision makers. The approach taken to achieve this goal is to make decisions at a global level which satisfy the constraints of local users. A global decision maker refers to the electricity provider, and it can be a national power company or a city-wide power company, for example. A local decision maker, on the other hand, is an electricity user, and can include an entire city's consumption or a particular sector of society (such as a hospital). For the remainder of this discussion, electric system decisions will take the form of adjusting the power output of generators which, by virtue of their physical connections, can provide power to various sectors (users) of society. Figure 1 shows an example electric grid in which this type of decision making may be necessary for providing power to a hospital, police department and fire department. The global decision maker (in this case, a city's utility company) must make a decision to provide power via any of the four generators to the three sectors. While it may seem obvious at first to operate only generator 2 (since it provides power to all sectors), further analysis may indicate that a combination of power to the other three generators may be optimal. This is due to the fact that one of the three sectors may be seen as more critical. This critical sector may require a greater certainty of power availability.

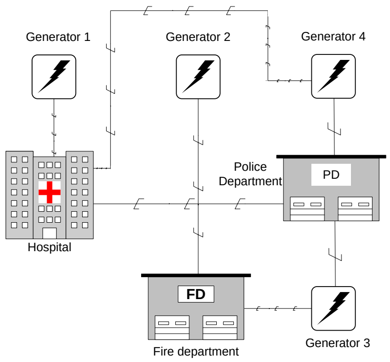

Figure 1: Electric Grid Illustration

In order to make electric system decisions as described above, this paper proposes the following problem formulation:

$$\min \text{Cost} = \sum_i C_i d_i$$
$$\text{s.t.} \left[\sum_i PP_{ij} d_i\right] - PD_j \geq 0 \qquad (1)$$

where i is the index of generators in the power system, C_i is the cost associated with the i^{th} generator, d_i is a decision variable indicating the power level associated with the i^{th} generator (defined on [0,1] where 0 indicates the generator is off and 1 indicates the generator is operating at 100% capacity), j is the index of sectors in the power system, PP_{ij} is the power provided by the i^{th} generator to the j^{th} sector, and PD_j is the total power demand of the j^{th} element in the system.

In Eq. (1), the objective is to minimize the global decision maker's (in this case, the electric company's) overall cost of power generation, given constraints on the required power imposed by sector users. This formulation ensures that both local and global demands are being met. While the optimal decision of Eq. (1) is not globally optimal (as the global decision maker's optimal cost is \$0 and the local decision makers would prefer to have all generators delivering power to their sector at 100%), it is a solution which is satisficing to all the involved decision makers. That is to say, the decision makers regard the solution as "good enough" while recognizing that it is not optimal for their own self interests [16]. This concept is essential when dealing with complex, interoperable systems. Sacrifices must always be made in order to obtain a solution that all involved decision makers find acceptable.

This formulation assumes complete certainty with regards to power provided and power demanded. This is not accurate in the context of a real world application. Therefore, the following formulation extends Eq. (1) to include uncertainty in PP_{ij} and PD_j:

$$\min \text{Cost} = \sum_i C_i d_i$$
$$\text{s.t.} P\left(\left[\sum_i PP_{ij} d_i\right] - PD_j \geq 0\right) \geq P_{crit_j} \qquad (2)$$

where P_{crit_j} is the criticality probability associated with the j^{th} sector, and all else is as before. Additionally, the constraint which refers to the power demand vs. the provided power is now defined probabilistically. That is, the net power must be delivered the j^{th} sector with a probability of at least P_{crit_j}. Cost is assumed to be deterministic for the purposes of this formulation.

This problem formulation is similar to the reliability-based design optimization problem formulation, which is discussed in the following section.

3. RELIABILITY-BASED DESIGN OPTIMIZATION

Reliability-based design optimization (RBDO) is concerned with finding a set of design variables for a given engineering system such that a given objective function (minimization of cost) is optimized and the design requirements (power demand) are satisfied with high probability. As mentioned earlier, the problem formulation for RBDO is the same as in Eq. (2). Within the probabilistic constraint, $\left[\sum_i PP_{ij} d_i\right] - PD_j$, which is generally denoted as $g_i(\ )$ and is referred to as a performance function in the RBDO literature, is formulated such that $g_i < 0$ indicates failure, $g_i > 0$ indicates success, and $g_i = 0$, the boundary between failure and success is referred to as the limit state.

There are two steps in solving Eq. (2). Step 1 is reliability analysis, i.e., evaluation of the probability constraint. Step 2 is optimization. Step 1 is discussed in detail below, focusing on a first-order approximation to calculate the probabilistic constraint in Eq. (2). Methods under step 2 are reviewed later in this section.

<u>Step1</u>: Analytical calculation of $P(g_i \leq 0\ )$ requires the evaluation of the integral of the joint probability density function (pdf) of all the random variables over the failure domain, as

$$P(g_i(d,x) \leq 0) = \int_{g_i(d,x)\leq 0} f_x(x)dx \qquad (3)$$

where d is the set of decision variables and x is the set of random variables.

This integral poses computational hurdles since it can be difficult to formulate the joint probability density explicitly and integration of a multidimensional integral may be difficult. Therefore, numerical integration methods such as Monte Carlo simulation or analytical approximations such as first-order reliability method (FORM) or second-order reliability method (SORM) are commonly used in mechanical systems reliability analysis. Monte Carlo simulation requires multiple runs of the deterministic system analysis and can be very costly. On the other hand, analytical approximations such as FORM and SORM are very efficient, and have been shown to provide reasonably accurate estimates of the probability integral for numerous applications in mechanical and structural systems. Detailed descriptions of these methods and computational issues are provided in [1, 5, and 7].

In FORM, the variables, x, which may each be of a different probability distribution, and may be correlated, are first translated to equivalent standard normal variables u. For uncorrelated normal variables, this transformation is simply $u_i = \dfrac{x_i - \mu_i}{\sigma_i}$. (Later, this concept is expanded to include variables that are non-normal and/or correlated). The limit state and the failure and safe regions are shown in Fig. 2, in the equivalent uncorrelated standard normal space u.

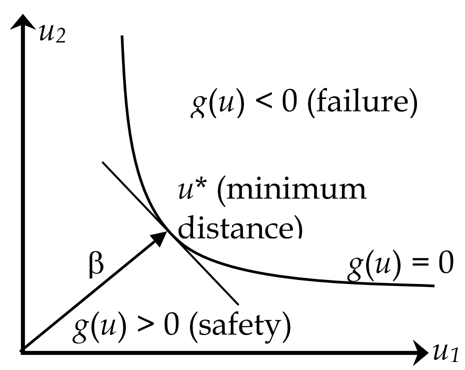

Figure 2: Illustration of limit state and failure and safe regions

The failure probability is now the integral of the joint normal pdf over the failure region. The FORM replaces the nonlinear boundary $g_i = 0$ with a linear approximation, at the closest point to the origin, and calculates the failure probability as

$$P_F = P(\, g_i(d,x) \le 0\,) = \Phi\,(-\beta_i) \qquad (4)$$

where P_F is the failure probability, Φ is the cumulative distribution function (CDF) of a standard normal variable and β_i is the minimum distance from the origin to the i^{th} limit state. Thus, the multidimensional integral in Eq. (3) is now approximated with a single dimensional integral as in Eq. (4), the argument of which (i.e., β_i) is calculated from a minimum distance search. The minimum distance point u^* on the limit state is also referred to as the most probable point (MPP), since linear approximation at this point gives the highest estimate of the failure probability as opposed to linearization at any other point on the limit state. (A second-order approximation of the failure boundary is referred to as SORM, where the failure probability calculation also requires curvatures of the limit state).

The minimum distance point (or MPP) u^* is found as the solution to the problem:

$$\min \beta_i \qquad (5)$$

$$\text{s.t. } g_i(d,x) \le 0$$

A Newton-based method to solve Eq. (5) was suggested by Rackwitz and Fiessler [13]. Other methods such as sequential quadratic programming (SQP) have also been used in the literature [6] and [19].

For non-normal variables, the transformation to uncorrelated standard normal space is $u_i = \dfrac{x_i - \mu_i^N}{\sigma_i^N}$, where μ_x^N and σ_x^N are the equivalent normal mean and standard deviation, respectively, of the x variables at each iteration during the minimum distance search. Rackwitz and Fiessler [13] suggested the solution of μ_x^N and σ_x^N by matching the PDF and CDF of the original variable and the equivalent normal variable at the iteration point. Other transformations are also available in [2, 11, 12, and 14].

If the variables are correlated, then the equivalent standard normals are also correlated. In that case, these are transformed to an uncorrelated space through an orthonormal transformation of the correlation matrix of the random variables through eigenvector analysis or a Cholesky factorization [7]. The minimum distance search and first-order or second-order approximation to the probability integral is then carried out in the uncorrelated standard normal space.

The minimum distance search typically involves five to ten evaluations of the limit state (and thus system analysis), and then the probability is evaluated using a simple analytical formula as in Eq. (4). Compared to this, Monte Carlo simulation may need thousands of samples if the failure probability is small, thus making Monte Carlo methods prohibitively expensive for solving large scale stochastic optimization problems.

Since the limit state functions involved in this problem formulation are linear in the random variables, and the random variables are assumed to be normal, FORM will be accurate. Second order estimates [3, 8, and 18] of the failure probability can also be used when the limit state is nonlinear, but due to the simplicity of the limit state function in this paper, second order methods are not found to be necessary.

The minimum distance point may also be found using a dual formulation of Eq. (5) as

$$\min\ g_i\ (d,x) \tag{6}$$

s.t. $\| u \| = \beta_{crit}$

This dual problem may be referred to as inverse FORM, and is used in the optimization (step 2) in this paper. In this formulation, β_{crit} is set to value corresponding to P_{crit}, as $\beta_{\text{crit}} = -\Phi^{-1}(P_{\text{crit}})$.

Step 2: In many implementations of reliability-based optimization, the probability constraint in Eq. (2) is usually replaced by a quantile equivalent, i.e., by a minimum distance constraint, as

$$\begin{aligned} &\min Cost(d) \\ &\text{s.t.}\, \beta_i \geq \beta_{\text{crit}} \end{aligned} \tag{7}$$

where β_i is the minimum distance computed from Eq. (5). Alternatively, the dual formulation has also been used, based on Eq. (6), as

$$\begin{aligned} &\min Cost(d) \\ &\text{s.t.}\, g_i(d,x) \geq 0 \end{aligned} \tag{8}$$

where g_i (d,x) is computed from Eq. (6).

Since the reliability constraint evaluation itself is an iterative procedure, the number of function evaluations required for reliability-based optimization is considerably larger than deterministic optimization. A simple nested implementation of RBDO (i.e., reliability analysis iterations nested within optimization iterations, as in Figure 3) tremendously increases the computational effort, and as a result, several approaches have been developed to improve the computational efficiency, typically measured in terms of the number of function evaluations required to reach a solution.

In decoupled methods [6, 15, and 19], the reliability analysis iterations and the optimization iterations are executed sequentially, instead of in a nested manner (refer to Figure 4, where OL means optimization loop and RL means reliability loop). This is done by fixing the results of one analysis while performing the iterations of the other analysis. Single loop methods [9, 10, and 17] perform the optimization through an equivalent deterministic formulation which replaces the reliability analysis constraint with the equivalent KKT condition at the minimum distance point on the limit state. Several versions of decoupled and single loops have been developed, based on whether direct or inverse FORM is used for the reliability analysis step. Note that FORM is the key to all these efficient RBDO techniques. Further information on the use of FORM in various RBDO formulations can be found in [4].

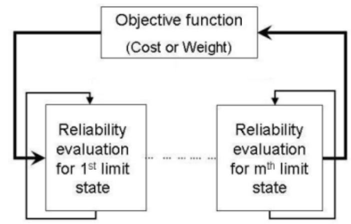

Figure 3: Illustration of Nested RBDO Method

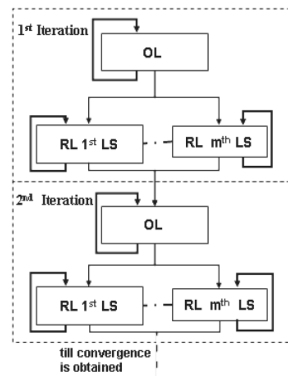

Figure 4: Illustration of Decoupled RBDO with Inverse FORM

For the purpose of cost optimization in this paper, a decoupled method of RBDO developed in [6] is chosen for reducing the number of function evaluations. In the cost optimization, a deterministic optimization is performed which determines a starting point for the reliability analysis. This deterministic optimization follows the formulation in Eq. (1) with the mean values of the design variables serving as the values for the random variables. This optimization yields a set of generator settings (d's). These setting are then passed to the reliability analysis. The reliability analysis then calculates the power demand and power provided (x's or most probable points, known as the MPPs) in the original variable space for the individual limit states, as outlined in Eq. (8). The MPP values are then fed back to the deterministic optimization and these values replace the random variable values (taken as the variable mean values in the first iteration). This process continues until convergence is reached on both a configuration and the MPP values. This process is outlined in Figure 5 below.

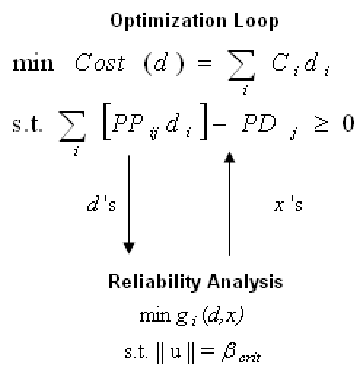

Figure 5: Decoupled Optimization Flowchart

The cost optimization is performed via the branch and bound method and the reliability analysis is performed via the SQP algorithm. The next section demonstrates this methodology on an example problem.

4. EXAMPLE PROBLEM

The example problem presented illustrates the problem formulation developed in Sections 2 and 3. The problem corresponds to the illustration shown in Figure 1. The generator data are as follows:

x_i	Cost	PP_{ij}		
		Hospital	Fire	Police
1	25	N(50,2.5)	-	-
2	15	N(5,0.25)	N(10,0.5)	N(5,0.25)
3	10	-	N(5,0.25)	N(15,0.75)
4	20	N(10,0.5)	-	N(20,1)

Table 1: Generator Data

The demand data for the sectors are as follows:

Sector	PD_j	P_{crit_j}
Hospital	N(50,2.5)	0.9
Fire	N(10,0.5)	0.75
Police	N(10,0.5)	0.75

Table 2: Demand Data

It should be noted that all random variables are normally distributed with a coefficient of variation (COV = μ/σ) of 5%.

The electric grid generator settings were optimized using four configurations: optimum (where generator settings could take on any value between zero and one) and integer-only deterministic variables (evaluated at the mean values of the variables), and optimum and integer-only stochastic variables. The generator setting results from the example problems are shown below:

	Stochastic		Deterministic	
Generator	Optimum	Integer Only	Optimum	Integer Only
1	1.00	1.00	0.92	1.00
2	0.86	1.00	0.80	1.00
3	0.37	1.00	0.40	1.00
4	0.03	0.00	0.00	0.00

Table 3: Generator Setting Results

The cost results from the example problems are shown below in Table 4.

	Cost
Stochastic Optimum	42.11
Stochastic Integer	50.00
Deterministic Optimum	39.00
Deterministic Integer	50.00

Table 4: Cost Results

It is obvious that the integer-only solutions are more expensive than the equivalent optimum solutions. This is due to the fact that integer solutions represent power configurations that are providing the sectors with excess power. Additionally, it makes sense that the optimum value for the stochastic scenario costs more money (i.e. requires more power) than its equivalent deterministic scenario. This is because excess power must be provided to ensure that the required demand is met with the specified P_{crit}. The stochastic and deterministic integer solutions result in the same settings because they both provide a level of excess power that is adequate in both the deterministic and stochastic scenarios.

5. CONCLUSIONS

Utilizing a first order reliability method, this paper developed an efficient methodology for making power system decisions at the global level that incorporates the needs of local power system users. This methodology includes consideration of uncertainty in power demand and provided power.

Several extensions should be explored in future power system decision making methodologies. They include:

- **Nonlinear power functions.** Delivered power and demand require more complicated modeling than is present in this methodology. While this methodology is an appropriate starting point, realistic models should be incorporated using RBDO.
- **Non-linear and non-deterministic costs.** Cost is assumed to be both linear (increasing as d_i increases)

and deterministic. The effects of both non-linear and stochastic costs should be investigated.

- **More complicated sector interactions.** Sectors in this paper do not have a direct influence on one another as they would in realistic scenarios (i.e. as one sector is powered, the other has decreased power delivered). These interactions should be investigated further and a more complicated interaction model should be developed.

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Biography

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